Sequential Bayesian methods for spatial on-line pore-pressure prediction from well log data

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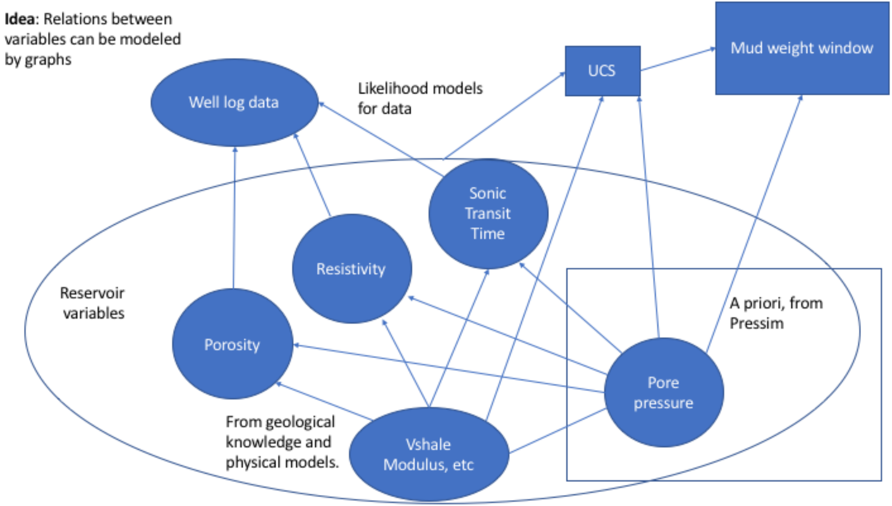
Graphical model



Main objective

Quantify uncertainty in pore pressure prediction using data assimilation method.

Problem description

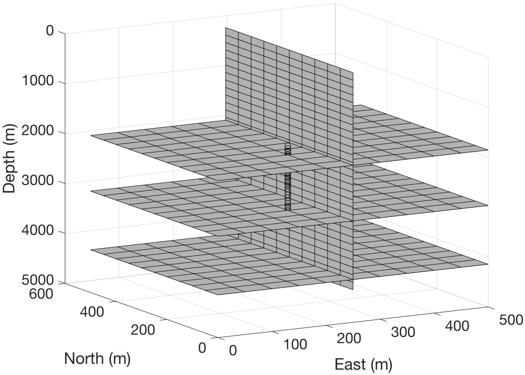
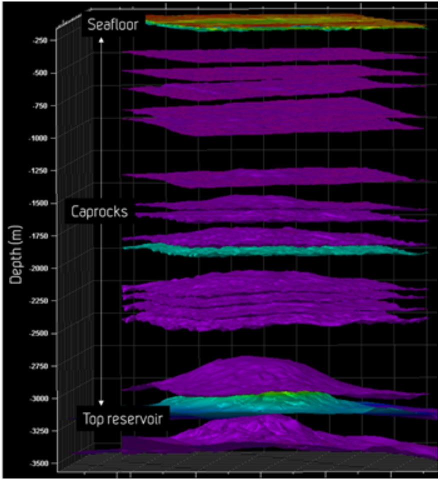


Figure: 3D grid of a subsurface domain

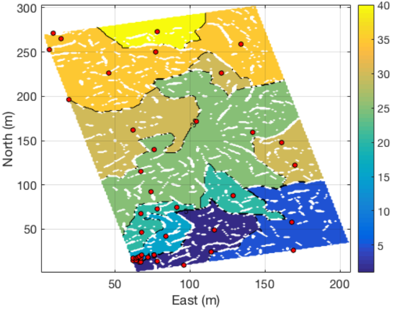
Workflow:

1. *train a prior distribution from realizations of pore pressure obtained from Pressim;*
2. *specify the likelihood model for well log data based on the available logs in the vicinity of the current location;*
3. *use a sequential updating method to get online pore pressure prediction.*

Prior model - geometry of data used to fit model



(a) Vertical view of the field showing the division in layers.



(b) Compartments separated by geological faults

Prior model - pore pressure realization

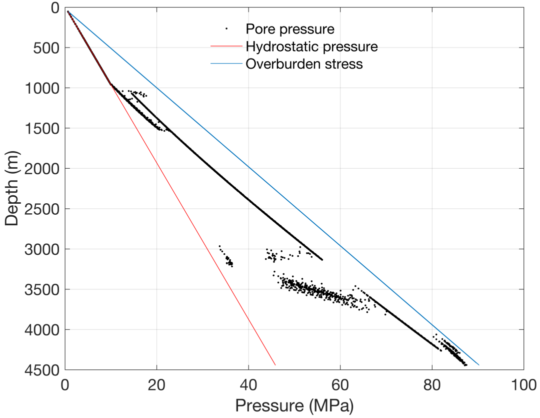


Figure: Pressures

Prior model - new variable

Constraint:

$$p_{h_i} < p_i < p_{ob_i}.$$

Logistic transform:

$$x_i = \log\left(\frac{p_i - p_{h_i}}{p_{ob_i} - p_i}\right)$$

$$\Rightarrow p_i = \frac{e^{x_i} p_{ob_i} + p_{h_i}}{1 + e^{x_i}}.$$

Prior model - linear regression

For each layer of the overpressured area we fitted a linear regression model

$$x_{i_k,k} = \beta_{0,k} + \beta_{1,k} s_{i_k 3,k} + \epsilon_{i_k,k} \quad k = 6, \dots, 18$$

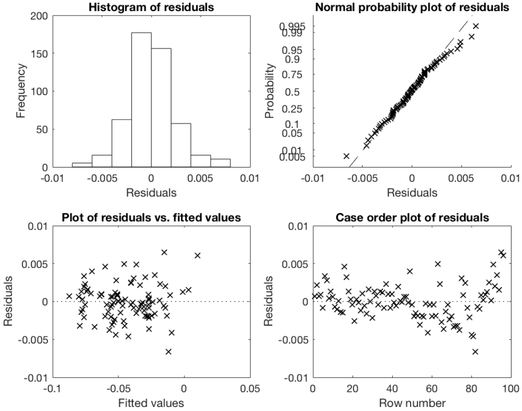
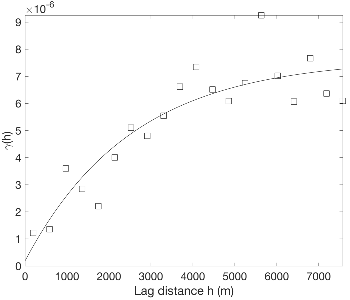
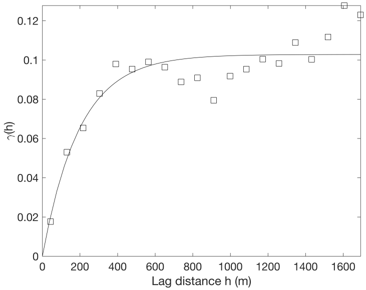


Figure: Residuals of the regression analysis for layer 8.

Prior model - fit variogram



(a) Empirical semivariogram (square) and fitted exponential semivariogram (solid line) for layer 8.



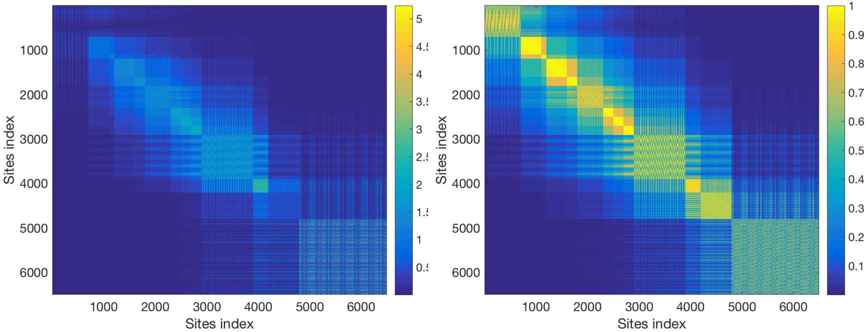
(b) Empirical semivariogram (square) and fitted exponential semivariogram (solid line) for compartment 41.

$$\gamma_k(h) = \sigma_k^2 \left(1 - \exp\left(-\frac{h}{r_k}\right)\right) \qquad k = 6, \ldots, 18,$$

$$\gamma_c(h) = \sigma_c^2 \left(1 - \exp\left(-\frac{h}{r_c}\right)\right) \qquad c = 1, \ldots, 41.$$

Prior model - prior covariance matrix

$$\boldsymbol{\Sigma}(\boldsymbol{s}_{i,k}, \boldsymbol{s}_{j,l}) = \sigma^2 \exp\left(-\frac{\sqrt{(s_{i1,k}-s_{j1,l})^2+(s_{i2,k}-s_{j2,l})^2}}{r_1} - \frac{|s_{i3,k}-s_{j3,l}|}{r_2}\right)$$



(a) Prior covariance

(b) Prior correlation

Prior model - mean

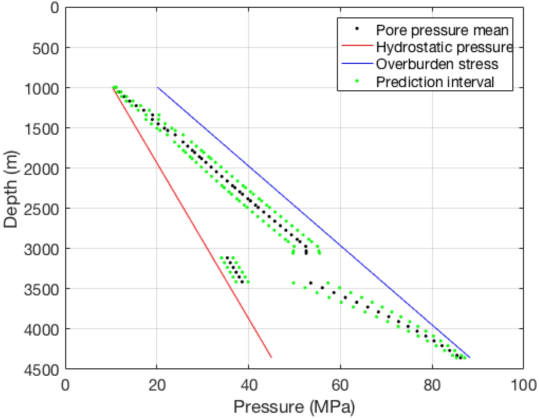


Figure: Pore pressure prior mean (black) with a 90% prediction interval (green)

Likelihood model - well log data

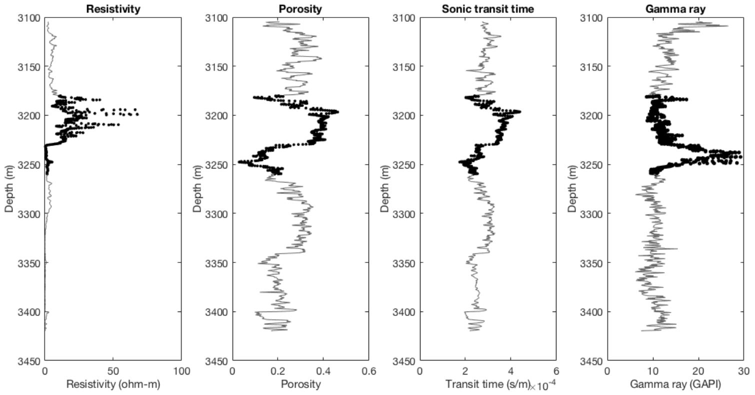


Figure: Well log measurements as function of the measured depth (MD). In the likelihood fitting the black dashed parts are ignored.

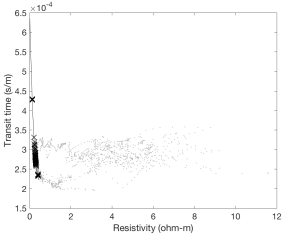
Likelihood model - rock physics relation [1]

$$\boldsymbol{y}_j = \begin{pmatrix} r_j \\ \phi_j \\ \Delta t_j \end{pmatrix} = \begin{pmatrix} \left(\frac{p_{ob_j} - p_j}{p_{ob_j} - p_{h_j}}\right)^{1/n_r} r_0 e^{bz_j} \\ \phi_0 \exp\left(-\frac{p_{ob_j} - p_j}{p_{ob_j} - p_{h_j}} c_\phi z_j\right) \\ (\Delta t_{ml} - \Delta t_m) \exp\left(\frac{p_j - p_{ob_j}}{p_{ob_j} - p_{h_j}} c_t z_j\right) + \Delta t_m \end{pmatrix} + \begin{pmatrix} \epsilon_{r_j} \\ \epsilon_{\phi_j} \\ \epsilon_{\Delta t_j} \end{pmatrix}$$

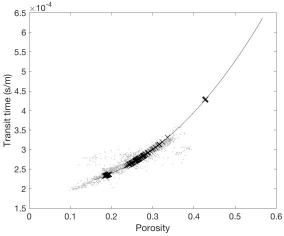
$$\Rightarrow \boldsymbol{y}_j = \boldsymbol{g}_j(p_j) + \boldsymbol{\epsilon}_j, \quad \boldsymbol{\epsilon}_j \sim N(0, \boldsymbol{R}), \qquad j = 1, ..., N$$

[1] Zhang J. Pore pressure prediction from well logs: methods, modifications, and new approaches. Earth Sci Rev 2011; 108:50–63.

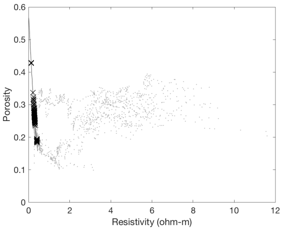
Likelihood model - paremeters values



(a) Transit time vs resistivity



(b) Transit time vs porosity



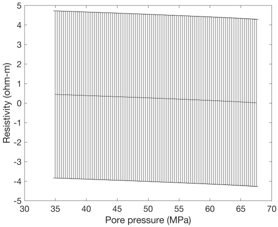
(c) Porosity vs resistivity

Likelihood model - measurements errors

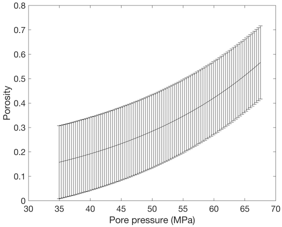
$$\hat{\boldsymbol{R}} = \frac{\sum_{j=1}^{N} \left(\boldsymbol{y}_j - \boldsymbol{g}_j(\mu_j)\right) \left(\boldsymbol{y}_j - \boldsymbol{g}_j(\mu_i)\right)^t}{N} - \left.\frac{\partial \boldsymbol{g}_j}{\partial p_j}\right|_{\boldsymbol{\mu}_j} Var(p_j) \left.\frac{\partial \boldsymbol{g}_j}{\partial p_j}\right|_{\boldsymbol{\mu}_j}^t$$

$$\hat{\boldsymbol{R}} = \begin{pmatrix} 6.8137 & 0.1503 & 8.5835 * 10^{-5} \\ 0.1503 & 0.0083 & 4.4036 * 10^{-6} \\ 8.5835 * 10^{-5} & 4.4036 * 10^{-6} & 2.5930 * 10^{-9} \end{pmatrix}.$$

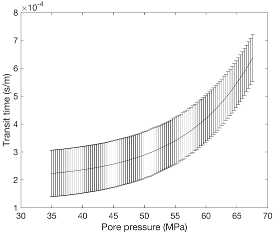
Likelihood model - model errors



(a) Resistivity



(b) Porosity



(c) Transit time

Sequential updating

Linearize likelihood at every step, leading to the Gaussian distribution $\pi(\boldsymbol{x}|\boldsymbol{y}_1, \dots, \boldsymbol{y}_j)$. The updated mean $\boldsymbol{m}_j = E(\boldsymbol{x}|\boldsymbol{y}_1, \dots, \boldsymbol{y}_j)$ and variance $\boldsymbol{V}_j = Var(\boldsymbol{x}|\boldsymbol{y}_1, \dots, \boldsymbol{y}_j)$ are computed recursively over the data gathering steps:

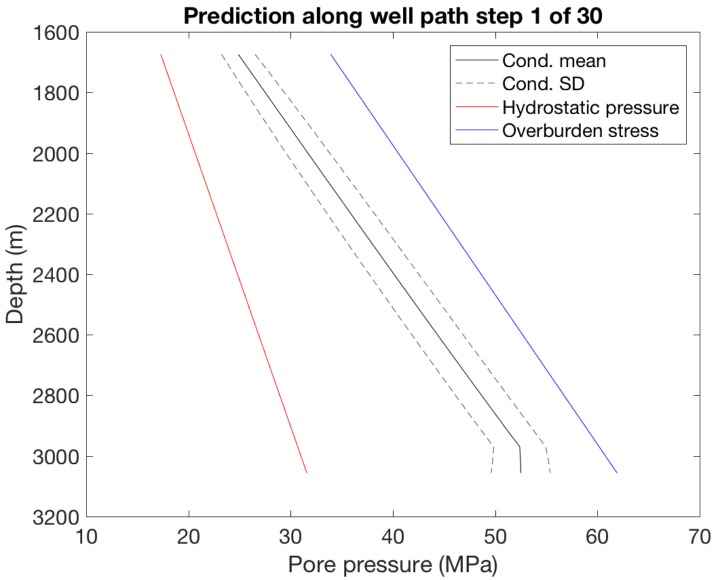
- Initialization:

$$\begin{aligned} \boldsymbol{m}_0 &= \boldsymbol{\mu}, \\ \boldsymbol{V}_0 &= \boldsymbol{\Sigma}, \end{aligned}$$

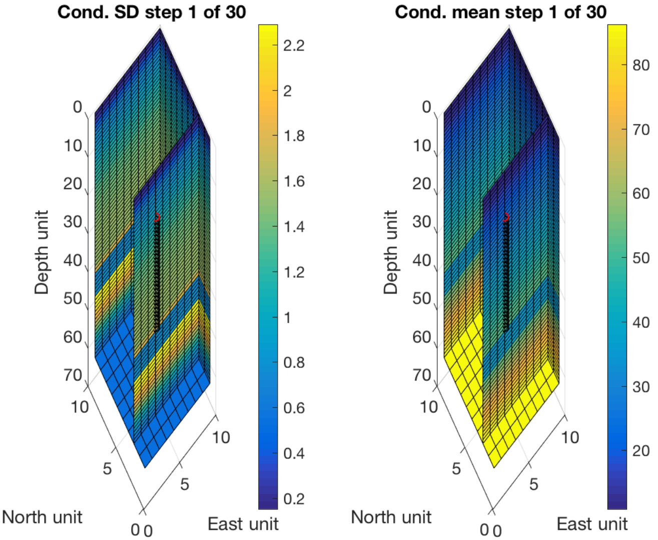
- Recursive updating for $j = 1, \dots, N$:

$$\begin{aligned} \boldsymbol{S}_j &= \boldsymbol{G}_j \boldsymbol{V}_{j-1} \boldsymbol{G}_j^t + \boldsymbol{R}, \\ \boldsymbol{K}_j &= \boldsymbol{V}_{j-1} \boldsymbol{G}_j^t \boldsymbol{S}_j^{-1}, \\ \boldsymbol{m}_j &= \boldsymbol{m}_{j-1} + \boldsymbol{K}_j (\boldsymbol{y}_j - \boldsymbol{g}_j(\boldsymbol{m}_{j-1})), \\ \boldsymbol{V}_j &= \boldsymbol{V}_{j-1} - \boldsymbol{K}_j \boldsymbol{G}_j \boldsymbol{V}_{j-1}. \end{aligned}$$

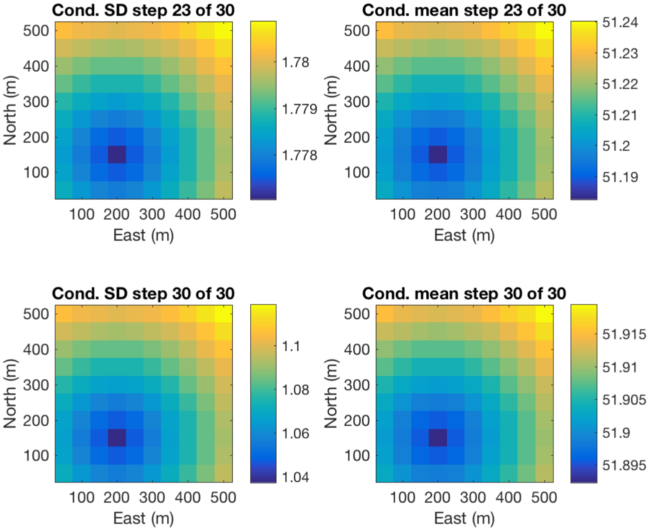
Results - along the well path



Results - updated mean and standard deviation



Results - updated mean and standard deviation



Sensitivity analysis - prior model

- Case I:
  $$\boldsymbol{\Sigma}_{new}(\boldsymbol{s}_{i,k}, \boldsymbol{s}_{j,l}) = \sigma^2_{new} \exp\left(-\frac{\sqrt{(s_{i1,k}-s_{j1,l})^2+(s_{i2,k}-s_{j2,l})^2}}{r_1} - \frac{|s_{i3,k}-s_{j3,l}|}{r_2}\right),$$
  with $\sigma^2_{new} = 2 * \sigma^2$
- Case II: $\boldsymbol{\Sigma}_{new} = \boldsymbol{\Sigma} + \boldsymbol{z}\boldsymbol{\Sigma}_{\beta_{gl}}\boldsymbol{z}^T$
- Case III: faults control lateral fluid flow.

SD values			
		143 m	0 m
Base case	Average SD	1.16	0.97
	SD at well site	1.47	0.93
Case I	Average SD	2.31	1.44
	SD at well site	2.28	1.37
Case II	Average SD	1.50	0.98
	SD at well site	1.48	0.95
Case III	Average SD	4.40	1.74
	SD at well site	4.39	1.65

Sensitivity analysis - measurements error

- Case IV: $\boldsymbol{R}_{new} = 4 * \boldsymbol{R}$
- Case V: $\boldsymbol{R}_{new} = \frac{1}{4} * \boldsymbol{R}$.

SD values			
		143 m	0 m
Case IV	Average SD	1.52	1.24
	SD at well site	1.51	1.22
Case V	Average SD	1.31	0.69
	SD at well site	1.27	0.65

Sensitivity analysis - data types

- Case VI: only resistivity
- Case VII: only porosity
- Case VIII: only sonic transit time
- Case IX: porosity and sonic transit time.

SD values			
		143 m	0 m
Case VI	Average SD	1.75	1.76
	SD at well site	1.75	1.76
Case VII	Average SD	1.70	1.45
	SD at well site	1.70	1.44
Case VIII	Average SD	1.75	1.75
	SD at well site	1.75	1.75
Case IX	Average SD	1.57	1.10
	SD at well site	1.56	1.05

Conclusion

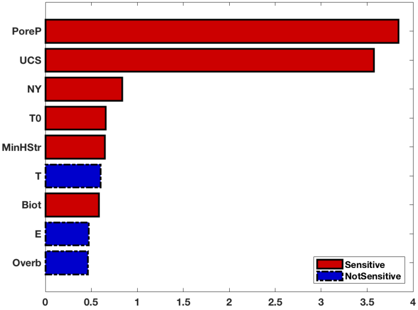
The main contribution of the study is pore pressure prediction highlighting the following points:

- **Bayesian modeling**: The approach provides consistent integration of pre-drill a priori knowledge about the pore pressure and the well log measurements.
- **Online**: The prediction of pore pressure is updated when the new well log data is available.
- **Spatial prediction**: The prediction is not only done near the borehole location, but also ahead of the bit and at other lateral and depth locations.
- **Uncertainty**: The spatial predictions of pore pressure are represented by a mean value best prediction and a variance/covariance description.

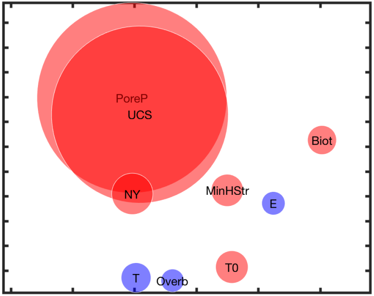
Acknowledgements

The work is carried out as a part of the KPN project 255418/E30: "Reduced uncertainty in overpressures and drilling window prediction ahead of the bit (PressureAhead)" and is funded by the Norwegian Research Council and the DrillWell Centre (AkerBP, Wintershall, ConocoPhillips and Statoil). ConocoPhillips have contributed with input data.

PSI - sensitivity analysis on mud weight window



(a) Main effects for upper bound



(b) Interactions for upper bound.